Completion of Matrix Inversions Using Elementary Matrix Inverse Multiplication Method

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Abstract.

Given a matrix *A* is ordo $m \times m$. Elementary matrix inverse multiplication (E_i) produces inverse matrix *A* which is $(E_n E_{n-1} E_{n-2} \ldots E_3 E_2 E_1) A = I$ where *I* is an identity matrix and $A^{-1} = E_n E_{n-1} E_{n-2} \ldots E_3 E_2 E_1$ is an inverse matrix *A*. The problem discussed in this research is to find a solution the ordo of matrices $m \times m$ by using inverse elementary matrix inversion method with the steps given in solving by completing the smallest ordo matrix first up until the ordo matrix $m \times m$.

Keywords: *Matrix, Inverse Matrix, Multiplication of Elementary Matrix.*

1. INTRODUCTION

A matrix is the arrangement of the right rectangles of numbers. The numbers in the arrangement are called entries or elements of matrix [1]. Matrices have *m* rows and *n* columns are expressed by [2]

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{m4} \end{bmatrix}$$

The point matrix has a fixed value of size [12]. The ordo of matrix is said to be an order determined by the number of rows and columns. If matrix *A* has *m* rows and *n* columns, then matrix *A* is ordered $m \times n$ [3]. A matrix has *m* rows and *n* columns can be expressed as $A_{m\times n} = (a_{ij})_{m\times n}$ [4].

In matrix calculations there are several operation matrices including addition, multiplication, determinant and inverse matrix. If *A* is a square matrix, and if there is matrix *B* of the same ordo such that $AB = BA = I$ then *A* is called invertible and *B* is inverse of *A* [5]. If the matrix *B* cannot be defined, then *A* is expressed as a singular matrix [6].

There are many methods can be used to determine the inverse of a matrix, including matrix partitioning, Gauss-Jordan elimination, adjoint methods and elementary matrix inverse multiplication methods. Determine of the inverse matrix is very influential from the ordo matrix. The greater of the ordo matrix so the higher of difficulty in determining inverse of a matrix. It is need a general form for determine the inverse of a matrix [7].

This research uses the elementary inverse multiplication method which is used to find the solutions of ordo matrix $m \times m$. In determining the inverse matrix using this method usually uses an intermediate matrix expressed by the matrix F_1. The matrix F_1 is the identity matrix where the *i*-th column element is replaced by the *i*-th column element of the square matrix *A* [1] and [8].

II. LITERATURE REVIEW

Definition 1. A matrix *A* has ordo $m \times m$ is called symmetric if $A^T = A$ [1]

Definition 2. The determinant matrix only defined on a square. The notation of determinant matrix A is: [2]

$$\det(A) = |A| \text{ or } \det A = |A| \tag{1}$$

Definition 3. If *A* is an ordo matrix $m \times m$ and if there is a matrix B has ordo $n \times n$ so that:

$$AB = BA = I \tag{2}$$

where *I* is the identity of ordo matrix $m \times m$, then matrix *A* is called non-singular and matrix *A* is the inverse of matrix *B* or matrix *B* is the inverse of matrix *A*. So the inverse of matrix *A* is written A^{-1} where [3]:

$$\begin{aligned} &AB = BA = I \\ &B = A^{-1}, A = B^{-1} \\ &AA^{-1} = A^{-1}A = I \end{aligned} \tag{3}$$

Definition 4. If matrix *A* is ordo $m \times m$. The elementary matrix inverse multiplication obtain an inverse matrix *A* [1] and [9]:

$$\left(E_n E_{n-1} E_{n-2} \ldots E_3 E_2 E_1\right) A = I \tag{4}$$

Definition 5. The matrix E_i is obtained from the transformation of identity matrix *I* that is the identity matrix where in *i* column is replaced by the normality of column vector or column matrix $\left(N_{k.i}\right)$ [10].

$$E_i = \begin{bmatrix} 1 & 0 & \ldots & N_{k.1} & \ldots & 0 \\ 0 & 1 & \ldots & N_{k.1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & N_{k.1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & N_{k.1} & \ldots & 1 \end{bmatrix} \tag{5}$$

Definition 6. The normality of column vectors or column matrices from ordo square matrices $n \times n$ for *i* columns replaced by $N_{k.i}$ is [10]:

$$N_{k.i} = \begin{bmatrix} \frac{-a_{1.i}}{a_{1.i}} \\ \vdots \\ \frac{-a_{(i-1).i}}{a_{i.i}} \\ \frac{1}{a_{i.i}} \\ \frac{-a_{(1+i).i}}{a_{i.i}} \\ \vdots \\ \frac{-a_{n.i}}{a_{i.i}} \end{bmatrix} \quad (6)$$

Definition 7. $a_{k.i}$ is an element of first column, where A_i is *i* column in matrix *A*. The value A_i is the *i* column in matrix *A*. For example A_i for the ordo 4×4 matrix [8] and [10]:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, A_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}, \ A_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{bmatrix}, \ A_3 = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{43} \end{bmatrix}, \ A_4 = \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \\ a_{44} \end{bmatrix}, A = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \end{bmatrix} \quad (7)$$

III. METHODS

The steps in this research are to solve the inverse matrix of the square matrix *A* using the inverse of the elementary inverse matrix:

1. Determine the F_1 matrix is replace the first column of the identity matrix I_n with A_1. The elementary inverse of F_1 is $(F_1)^{-1} = E_1 I = E_1$
2. Determine the F_2 matrix is replace the second column of the F_1 matrix with A_1. Inverse of F_2 is $(F_2)^{-1} = E_2 (F_1)^{-1} = E_2 E_1$
3. Determine the F_3 matrix is replace the third column of the F_2 matrix with A_3. The inverse of F_3 is $(F_3)^{-1} = E_3 (F_2)^{-1} = E_3 E_2 E_1$
4. This steps is carried out until all column vectors *A* are entered into the matrix $F_1 = A$ or $i = n$, the inverse of *A* will be obtained $A^{-1} = (F_i)^{-1} = E_i ... E_3 E_2 E_1$
5. The steps is complete and an inverse matrix is obtained with the ordo given.

IV. RESULT AND DISCUSSION

Result and discussion in this research using some examples from ordo matrices $m \times m$

1. The elementary inverse matrix ordo 3 *x* 3

Solving the matrix $A = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

The step 1: looking for inverse matrix F_1^{-1}

$$A = \begin{bmatrix} A_1 & A_2 & A_1 \end{bmatrix}, A_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, A_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, F_1 = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$a_{k.1} = IA_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad N_{k.1} = \begin{bmatrix} \frac{1}{a_{11}} \\ \frac{-a_{21}}{a_{11}} \\ \frac{-a_{31}}{a_{11}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{2}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \\ -0.5 \end{bmatrix}$$

$$F_1 : \left(F_1^{-1}\right) = E_1 = \begin{bmatrix} N_{11} & 0 & 0 \\ N_{21} & 1 & 0 \\ N_{31} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ -1 & 1 & 0 \\ -0.5 & 0 & 1 \end{bmatrix}$$

The step 2: looking for invers matrix F_2^{-1}

$$F_2 = \begin{bmatrix} 2 & 3 & 0 \\ 2 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \quad a_{k.2} = \left(F_1^{-1}\right) A_2 = E_1 A_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ -1 & 1 & 0 \\ -0.5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ -1 \\ 0.5 \end{bmatrix}$$

$$N_{k.2} = \begin{bmatrix} -\dfrac{a_{12}}{a_{22}} \\ \dfrac{1}{a_{22}} \\ -\dfrac{a_{32}}{a_{22}} \end{bmatrix} = \begin{bmatrix} -\dfrac{1.5}{(-1)} \\ \dfrac{1}{-1} \\ -\dfrac{0.5}{(-1)} \end{bmatrix} = \begin{bmatrix} 1.5 \\ -1 \\ 0.5 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & N_{12} & 0 \\ 0 & N_{22} & 0 \\ 0 & N_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1.5 & 0 \\ 0 & -1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$$

Invers dari $\left(F_2^{-1}\right) = E_2 E_1 = \begin{bmatrix} 1 & 1.5 & 0 \\ 0 & -1 & 0 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ -1 & 1 & 0 \\ -0.5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1.5 & 0 \\ 1 & -1 & 0 \\ -1 & 0.5 & 1 \end{bmatrix}$

The step 3: looking for invers matrix F_3^{-1}

$$F_3 = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}, \quad a_{k.3} = \left(F_2^{-1}\right) A_3 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1 \\ 0.5 \end{bmatrix}$$

$$N_{k.3} = \begin{bmatrix} -\dfrac{a_{13}}{a_{33}} \\ -\dfrac{a_{23}}{a_{33}} \\ -\dfrac{1}{a_{33}} \end{bmatrix} = \begin{bmatrix} -\dfrac{-(0.5)}{(0.5)} \\ \dfrac{-1}{0.5} \\ -\dfrac{1}{0.5} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & N_{13} \\ 0 & 1 & N_{23} \\ 0 & 0 & N_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\left(F_3^{-1}\right) = E_3 \left(F_2\right)^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1.5 & 0 \\ 1 & -1 & 0 \\ -1 & 0.5 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 1 \\ 3 & -2 & -2 \\ -2 & 1 & 2 \end{bmatrix}$$

Inverse $A^{-1} = E_3 E_2 E_1 = E_3 \left(F_2\right)^{-1} = \begin{bmatrix} -2 & 2 & 1 \\ 3 & -2 & -2 \\ -2 & 1 & 2 \end{bmatrix}$

2. The elementary inverse matrix ordo 4 *x* 4

Solving the matrix $A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 4 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix}$

The step 1: looking for inverse matrix F_1^{-1}

$$A=\begin{bmatrix}A_1 & A_2 & A_3 & A_4\end{bmatrix}, A_1=\begin{bmatrix}2\\4\\1\\2\end{bmatrix},\ A_2=\begin{bmatrix}1\\1\\2\\1\end{bmatrix},\ A_3=\begin{bmatrix}2\\2\\2\\1\end{bmatrix},\ A_4=\begin{bmatrix}1\\2\\2\\2\end{bmatrix}, F_1=\begin{bmatrix}2&0&0&0\\4&1&0&0\\1&0&1&0\\2&0&0&1\end{bmatrix}$$

$$a_{k.1}=IA_1\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}2\\4\\1\\2\end{bmatrix}=\begin{bmatrix}2\\4\\1\\2\end{bmatrix},\quad N_{k.1}=\begin{bmatrix}\frac{1}{a_{11}}\\\frac{-a_{11}}{a_{11}}\\\frac{-a_{31}}{a_{11}}\\\frac{-a_{41}}{a_{11}}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\\frac{-4}{2}\\\frac{-1}{2}\\\frac{-2}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\-2\\-\frac{1}{2}\\-1\end{bmatrix}, E_1=\begin{bmatrix}N_{11}&0&0&0\\N_{21}&1&0&0\\N_{31}&0&1&0\\N_{41}&0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0&0\\-2&1&0&0\\-\frac{1}{2}&0&1&0\\-1&0&0&1\end{bmatrix}$$

Inverse $F_1:(F_1)^{-1}=E_1=\begin{bmatrix}\frac{1}{2}&0&0&0\\-2&1&0&0\\-\frac{1}{2}&0&1&0\\-1&0&0&1\end{bmatrix}$

The Step 2: looking for inverse matrix F_2^{-1}

$$F_2=\begin{bmatrix}2&1&0&0\\4&1&0&0\\1&2&1&0\\2&1&0&1\end{bmatrix},\ a_{k.2}=\left(F_1^{-1}\right)A_2=E_1A_2=\begin{bmatrix}\frac{1}{2}&0&0&0\\-2&1&0&0\\-\frac{1}{2}&0&1&0\\-1&0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\2\\1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\-1\\\frac{3}{2}\\0\end{bmatrix}$$

$$N_{k.2}=\begin{bmatrix}\frac{-a_{12}}{a_{22}}\\\frac{1}{a_{22}}\\\frac{-a_{32}}{a_{22}}\\\frac{-a_{42}}{a_{22}}\end{bmatrix}=\begin{bmatrix}\frac{-1/2}{-1}\\\frac{1}{-1}\\\frac{-3/2}{-1}\\\frac{-0}{-1}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\-1\\\frac{3}{2}\\0\end{bmatrix},\quad E_2=\begin{bmatrix}1&N_{12}&0&0\\0&N_{22}&0&0\\N_{31}&N_{32}&1&0\\N_{41}&N_{42}&0&1\end{bmatrix}=\begin{bmatrix}1&\frac{1}{2}&0&0\\0&-1&0&0\\0&\frac{3}{2}&1&0\\0&0&0&1\end{bmatrix}$$

Inverse $$(F_2)^{-1} = E_2E_1 = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & -1 & 0 & 0 \\ -\frac{7}{2} & \frac{3}{2} & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

The Step 3: looking for inverse matrix F_3^{-1}

$$F_3 = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 4 & 1 & 2 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 1 & 1 & 1 \end{bmatrix}, \quad a_{k.3} = \left(F_2^{-1}\right)A_3 = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & -1 & 0 & 0 \\ -\frac{7}{2} & \frac{3}{2} & 1 & 0 \\ -12 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ -1 \end{bmatrix}$$

$$N_{k.3} = \begin{bmatrix} \frac{-a_{13}}{a_{33}} \\ \frac{-a_{23}}{a_{33}} \\ \frac{1}{a_{33}} \\ \frac{-a_{43}}{a_{33}} \end{bmatrix} = \begin{bmatrix} \frac{-0}{-2} \\ \frac{-2}{-2} \\ \frac{1}{-2} \\ \frac{-(1)}{-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & N_{13} & 0 \\ 0 & 1 & N_{23} & 0 \\ 0 & 0 & N_{33} & 0 \\ 0 & 0 & N_{43} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$(F_3)^{-1} = E_3(F_3)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 2 & -1 & 1 & 0 \\ -\frac{7}{2} & \frac{3}{2} & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \quad (F_3)^{-1} = E_3(F_3)^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & \frac{1}{2} & 1 & 0 \\ \frac{7}{4} & -\frac{3}{4} & -\frac{1}{2} & 0 \\ \frac{3}{4} & -\frac{3}{4} & -\frac{1}{2} & 1 \end{bmatrix}$$

The Step 4: looking for inverse matrix F_3^{-1}

$$F_3 = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 4 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix}, \; a_{k.4} = \left(F_3^{-1}\right) A_4 = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & \frac{1}{2} & 1 & 0 \\ \frac{7}{4} & -\frac{3}{4} & -\frac{1}{2} & 0 \\ \frac{3}{4} & -\frac{3}{4} & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -\frac{3}{4} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.5 \\ -0.75 \\ 0.25 \end{bmatrix}$$

$$N_{k.4} = \begin{bmatrix} \frac{-a_{14}}{a_{44}} \\ \frac{-a_{24}}{a_{44}} \\ \frac{-a_{34}}{a_{44}} \\ \frac{1}{a_{44}} \end{bmatrix} = \begin{bmatrix} \frac{-0.5}{0.25} \\ \frac{-1.5}{0.25} \\ \frac{0.75}{0.25} \\ \frac{1}{0.25} \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \\ 3 \\ 4 \end{bmatrix}, \; E_3 = \begin{bmatrix} 1 & 0 & 0 & N_{14} \\ 0 & 1 & 0 & N_{24} \\ 0 & 0 & 1 & N_{34} \\ 0 & 0 & 0 & N_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\text{Inverse } A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

V. CONCLUSION

After doing the calculation to obtain the inverse of ordo $m \times m$ matrix such as 3×3, 4×4 using the inverse of elementary matrix inverse method is obtained. Further research with the same research with the help of mathematical applications such as Microsoft. Excel, MATLAB and others to obtain an inverse matrix and the other matrix compositions with orde $m \times m$ can be done.

REFERENCES

[1] A. Howard, C. Rorres, **Elementary Linear Algebra, Ninth Edition**, John Wiley & Sons, Inc. All Rights Reserved, 2014.
[2] S. Wolfram, *The Mathematica Book*, Wolfram Media, 5th edition, 2003.

[3] S. Wagon, *Mathematica in Action*, Springer, 3rd edition, 2010.
[4] S, Jain, A. D. Gunawardena, **Linear Algebra An Interactive Approch**, Unite States of America, 2004.
[5] F. C, Chang, *Determinant Of Matrix By Order Condensation*. **British J. of Math. & Comput. Science, 13**, 2014, pp. 1843-1848.
[6] J, Sherman, W.J, Morrison. *Adjustment Of Inverse Matrix Corresponding To Changes In A Given Column Or A Given Row Of The Original Matrix*. **Ann. Math. Stat, 75**, 1949, pp. 124.
[7] C. T, Su, F. C, Chang, *Quick Evaluation Of Determinant*. **Appl. Math. & Comput, 75**, 1996, pp. 117-118.
[8] K. Aydin, C, Kizilkan, *Iterative Inverse Algorithm For Perturbed Matrix*. **SDU Science Journa**l, **3**, 2008, pp. 107-112.
[9] J. Rohn, *Inverse interval matrix*. **SIAM Journal on Numerical Analysis, 30**, 1993, pp. 864–870.
[10] F. C. Chang, *Inverse And Determinant Of A Square Matrix By Order Expansion And Condensation*. **IEEE Antenna and Propagation Magazine, 57**, 2015, pp. 28-32.
[11] A. Asif , J. Moura, *Block Matrices With L-Block-Banded Inverse: Inversion Algorithm*. **IEEE Trans. on Signal Processing, 53**, 2005, pp. 630-642.
[12] C. Jianshu, X, Wang, *New Recursive Algorithm For Matrix Inversion*, **J. Systems Engineering and Electronics, 19**, 2008, pp. 381-384.