Relative Predictive Accuracy of Machine Learning-Enhanced Long Memory Volatility Models for Modeling Nigeria Energy Data

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***Abstract*.**

This study investigates the relative predictive efficacy of machine learning-augmented long memory volatility models in analyzing Nigeria's energy statistics from January 1960 to December 2024. Utilizing monthly energy pricing information sourced from the Central Bank of Nigeria, the research employs thorough econometric and computational methods to evaluate the persistence of volatility, significant structural changes, and long-term dependencies existing within the energy data of Nigeria. Initial assessments show that raw energy data display non-stationarity, structural volatility, and clustering phenomena, with the Augmented Dickey-Fuller, Phillips-Perron, and KPSS tests affirming non-stationarity in the level data and stationarity in the returns data. The analysis of structural breaks utilizing the ruptures algorithm discovers eight notable breakpoints that align with key policy changes, worldwide oil crises, and organizational transformations in Nigeria's energy sector. To represent the noted persistence and long memory, established econometric models such as ARFIMA, FIGARCH, and HYGARCH are estimated and then combined with Artificial Neural Networks (ANN) and Support Vector Regression (SVR). The outcomes demonstrate that hybrid models significantly exceed the performance of their isolated versions, with ARFIMA–ANN and FIGARCH–ANN showing the lowest Mean Squared Errors at 0.034 and 0.035 respectively. The ANN consistently shows a greater capability to capture nonlinear volatility patterns, while SVR demonstrates a moderate level of success. The results highlight that integrating long-memory stochastic models with machine learning frameworks provides strong predictive performance for complex energy series that depend on different regimes. These findings have significant consequences for the formulation of energy policies, management of volatility, and investment strategies in Nigeria's transforming energy sector. The study concludes that econometric models enhanced by machine learning are vital for developing adaptable forecasting systems in emerging markets facing structural and policy changes.

Keywords: *Volatility Models; Long Memory; Machine Learning; Energy Data and Structural Break.*

I. INTRODUCTION

The rising difficulty and volatility of energy markets in Nigeria highlight the pressing need for more effective and precise forecasting methods. The fluctuations in energy prices are crucial to Nigeria's economic framework, as they heavily impact economic planning, investment strategies, and overall economic health. Classical econometric approaches, including ARIMA and GARCH models as commonly used, frequently fail to account for the long-memory and non-linear traits found in financial and energy time series data.
Current research indicates a heightened interest in long-memory devices and machine learning (ML) approaches for evaluating economic and financial metrics in Nigeria. Deebom, Etuk, and Nwikorgah (2021) revealed long-memory attributes in return innovations from growing agricultural markets, pointing out the limitations of short-memory models in these scenarios. Likewise, Tuaneh et al. (2025) expanded this exploration to the lending rates of Nigerian commercial banks by using ARIMA, ARFIMA, and FIGARCH, assessing their effectiveness in capturing enduring dependencies. These investigations emphasize the importance of modeling long-memory volatility across multiple financial sectors in Nigeria.Conversely, the use of machine learning in financial forecasting has been gaining traction. Ogundunmade, Adepoju, and Allam (2022) applied ML techniques to anticipate crude oil prices in Nigeria, demonstrating their potential advantages over traditional time series methodologies in managing intricate, non-linear data formats.

David et al. (2024) offered a comprehensive review of ML applications in stock market predictions, showing marked enhancements in forecasting accuracy across various models. Adams and Uchema (2024) further analyzed the performance of LSTM (a deep learning model) as opposed to EGARCH in forecasting exchange rate fluctuations, illustrating that ML approaches can adeptly recognize dynamic trends in environments characterized by high volatility.Nevertheless, a significant gap exists in merging long-memory volatility models with advancements in machine learning, particularly within Nigeria's energy sector. Previous research has primarily centered on either conventional long-memory models (such as ARFIMA, FIGARCH) or isolated ML strategies, and there is a scarcity of empirical data on the comparative predictive accuracy of machine learning-augmented long-memory volatility models for Nigeria's energy metrics. The integration of these strategies could provide a more sophisticated insight into volatility behaviors, enhance forecasting reliability, and better guide policy and investment actions.Therefore, this research aims to address this gap by assessing the comparative predictive efficacy of machine learning-enhanced long-memory volatility models in analyzing Nigeria's energy data. By pursuing this objective, it intends to add to the expanding literature at the intersection of econometrics and machine learning, while tackling the shortcomings of current forecasting frameworks in reflecting the intricate, long-term dependencies that are characteristic of energy markets.

II. METHODS

The research utilizes a quantitative longitudinal (time-series) methodology by leveraging monthly energy statistics from Nigeria spanning the years 1960, January to 2024, December. The software used for the data analysis is python 3.11. The process of data analysis commenced with gathering energy-related data from Nigeria. This encompassed metrics regarding energy generation, usage, and price indices that were recorded over an extended period. Initially, the data, presumed to be in its raw state, was depicted as a time series to examine the overall trend, seasonal patterns, and variability. This visual representation offered an initial perspective on the characteristics of the series and indicated possible non-stationarity due to observable trends and fluctuations throughout the period. Following Deebom et al (2021) approach the raw energy data underwent transformation to create two derived return series: simple returns and logarithmic returns. Simple returns were determined by calculating the first difference normalized by the preceding observation of the raw data, while logarithmic returns were derived from the differences in the natural logarithm of consecutive data points (Deebom et al, 2021). These changes aimed to stabilize the variance and potentially attain stationarity. The time plots for the transformed series were subsequently examined, revealing traits like volatility clustering and variations around a stable mean, indicating a more stationary nature compared to the original data. To statistically validate the visual findings, descriptive statistics were computed for all three series:

Simple returns are given as

$$(RE_t) = \frac{RE_t - RE_{t-1}}{RE_{t-1}} \qquad (1)$$

$$\text{Log Returns:}(InRE_t) = In \quad (\frac{RE_t - RE_{t-1}}{RE_{t-1}}) * 100 \quad (2)$$

The metrics to be measured include mean, median, standard deviation, skewness, and kurtosis were calculated and outlined. These statistical summaries shed light on the central tendency, range, and distribution characteristics of the dataset. Additionally, the Jarque-Bera normality test was performed to assess deviations from normality, showing substantial departures across all three series, particularly due to pronounced skewness and leptokurtosis in the return series. After completing the descriptive analysis, structural changes within the energy series were detected using the "ruptures" package, a contemporary change point detection method available in Python. This approach systematically identified points within the series where statistical characteristics shifted, hinting at possible changes in policy, regulatory modifications, or economic disturbances. The specific dates of these changes will be estimated, while the corresponding time plot visually illustrates these interruptions in the energy time series. The identified breakpoints were then associated with significant historical and institutional occurrences within Nigeria's energy sector, enhancing the contextual comprehension of the dynamics involved (Deebom et al, 2021).The subsequent phase included the tests for stationarity through three complementary methods: the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

These assessments were applied to each of the three series, with the findings being interpreted together to prevent dependence on just one testing approach (Tuaneh et al (2025). The raw energy series did not pass either the ADF or PP tests (indicating it was non-stationary) and was validated as non-stationary by the KPSS test. Conversely, both return series successfully passed the stationarity assessments, confirming their applicability in subsequent modeling efforts.Next, the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots were created for all three series to evaluate serial dependence and assist in determining potential model specifications. For the raw series, the Autocorrelation Function (ACF) exhibited a gradual reduction, characteristic of non-stationarity. In contrast, the ACF and Partial Autocorrelation Function (PACF) visuals for the return series showed a rapid decline after a few lags, signifying stationarity and appropriateness for simplified ARMA models. These visuals and proper identification were crucial for discerning short-term memory patterns and offered direction for initial model choices, including AR(1), MA(1), or ARMA(1,1).To examine the variability in volatility, which is prevalent in financial and energy series, tests for Autoregressive Conditional Heteroskedasticity (ARCH) effects were carried out using the Lagrange Multiplier (LM) test.

The findings presented in Table 3 demonstrated notable ARCH effects in the original energy data across all tested lags (5, 10, 15), confirming the occurrence of volatility clustering. However, no significant ARCH effects were found within the return series, particularly in simple returns, suggesting that while the raw data necessitated GARCH-type volatility modeling, the return series might not require such methods.Further analysis centered on assessing long memory characteristics within the dataset. The Hurst Exponent, Detrended Fluctuation Analysis (DFA) alpha, Rescaled Range statistics, and Variance Ratio tests were calculated to identify enduring autocorrelations over extended periods. These results underscored the necessity for models that incorporate memory effects in the raw series, while standard time series models were deemed adequate for the return data (Tuaneh et al, 2025 and Deebom et al, 2023).Simultaneously, the residuals from the return series were scrutinized for squared autocorrelations. Observational assessments and additional tests indicated that, despite the observed stationarity, some patterns in the residuals might persist, suggesting that GARCH-family models could still be relevant for capturing higher-order relationships or volatility clustering in return under specific conditions. Also, this strategy merges econometric analysis with supervised machine learning techniques to account for both linear and nonlinear fluctuations in volatility. The econometric frameworks employed include the ARFIMA, FIGARCH, and HYGARCH models. The formulation of the ARFIMA model is defined as:

$$(1-\sum_{i=1}^{\rho}\theta_i L^i)^d(1-L)^d Y_t = (1+\sum_{j=1}^{q}\theta_j L^j)\varepsilon_t, \varepsilon_t \approx (0,\sigma_t^2) \quad (3)$$

where Y_t is the observed scalar time series at time t (here: energy price level or transformed series). L represents the lag / backshift operator such that$L^K Y_t = Y_{t-k}$. Polynomials in L implement autoregressive and moving-average lags. Also, ρ is the autoregressive order (non-negative integer). The AR polynomial has terms up to lag ρ while q is the moving-average order (non-negative integer). The MA polynomial has terms up to lag q. Similarly, θ_i for i=1,…,ρi=1, Autoregressive coefficients (AR parameters). They scale the lagged valuesY_{t-1} inside the fractional AR operator. In your original notation these were θ_j ; here we use θ_j to distinguish them from the MA coefficients. θ_j for j=1…,qj=1, Moving-average coefficients (MA parameters). They multiply past shocks ε_{t-j} and d is the fractional differencing / long-memory parameter while the "d" is said to lies $0 < d < 1$ and this captures the long-memory parameter.

Similarly, the FIGARCH model is given as; $\sigma_t^2 = \omega + [1-\beta(L)-(1-\varphi(L))(1-L)^d\varepsilon_t^2$

where σ_t^2 is the conditional variance of ε_t at time t. ω>0 constant term (long-run variance anchor). Must be nonnegative; commonly strictly positive to avoid degenerate variance. Also, βi (i=1,…,pi) are coefficients of the lag polynomial β(L). They govern autoregressive persistence in variance (like GARCH β terms). φi(i=1,…,ri=1,): coefficients of the lag polynomial φ(L). They appear in the fractionally integrated operator affecting how past squared shocks feed into variance. d∈[0,1) is the fractional integration parameter for the variance process. Also, d=0→ reduces to a standard (possibly GARCH-like) model (no long memory). Also, 0<d<1 → long memory in volatility: hyperbolic decay of the impulse-response of shocks to conditional variance and d close to 0.5 often indicates strong persistence but possible infinite unconditional variance

(depends on other coefficients). In another development, the HYGARCH Model used in the study is given as: $\sigma_t^2 = (1-\lambda)\omega + \lambda(1-L)^d\omega + [1-\beta(L)-(1-\varphi(L))(1-L)^d]\varepsilon_t^2$
where λ regulates hyperbolic decay in volatility persistence. When λ=0, the HYGARCH collapses to a standard GARCH model (short-memory volatility). When λ=1, it reduces to a FIGARCH model (pure long-memory volatility). For 0<λ<1, the model exhibits hyperbolic decay — a smoother and more flexible transition between short and long memory, capturing both transitory and persistent volatility effects. This flexibility allows the HYGARCH to retain long memory features while maintaining a finite unconditional variance (a limitation of FIGARCH).

In the context of this study, the Hybrid (Machine Learning–Enhanced) models used residual values from the ARFIMA/FIGARCH models are used as inputs into machine learning frameworks such as the ARFIMA–ANN Hybrid and ARFIMA–SVR Hybrid. The ARFIMA–ANN Hybrid is given as $\hat{Y}_t = f(y_{t-1}.y_{r-2}..........w, b)$,where f (·) is a multilayer perceptron minimizing MSE. Also, ARFIMA–SVR Hybrid is given as:

$$\hat{Y}_t = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)K(X_i, X_t) + b \quad (4)$$

where K(·) is the radial basis function kernel, $\hat{Y}_t$ is predicted value of the residuals or the final forecast, Xi and Xt= input vectors (training and test data points, respectively), αi and ,αi∗ represents Lagrange multipliers obtained from the SVR optimization problem, K(Xi,Xt) is the kernel function (e.g., linear, polynomial, radial basis function (RBF), or sigmoid), and b is bias term or intercept. The residuals ε_t are then modeled using Support Vector Regression, which estimates the nonlinear patterns through the kernel function as shown:

$$\varepsilon_t = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)K(X_i, X_t) + b \qquad (5)$$

The traditional econometric models are assessed through maximum likelihood and training with machine learning using backpropagation (ANN) and sequential minimal optimization (SVR). The Final hybrid prediction is achieved through the integration of the linear estimations from ARFIMA and the nonlinear segment from SVR is given as : $\hat{Y}_t^{Hybride} = \hat{Y}_t^{ARFIMA} + \hat{\varepsilon}_t^{\text{SVR}}$
Therefore, the combination of the ARFIMA and SVR models effectively merges the fractional differencing characteristic of ARFIMA for handling long-memory patterns with the nonlinear mapping abilities of SVR, resulting in enhanced forecasting accuracy for intricate, nonstationary time series data. The metrics for performance comprise Akaike Information Criterion (AIC), Mean Squared Error (MSE), and a comparison of forecast errors over a 12-month period. The findings demonstrate that ARFIMA–ANN attained the highest predictive capability, confirming the effectiveness of hybrid long-memory machine learning systems for analyzing Nigeria's energy fluctuations.

III. RESULT AND DISCUSSION

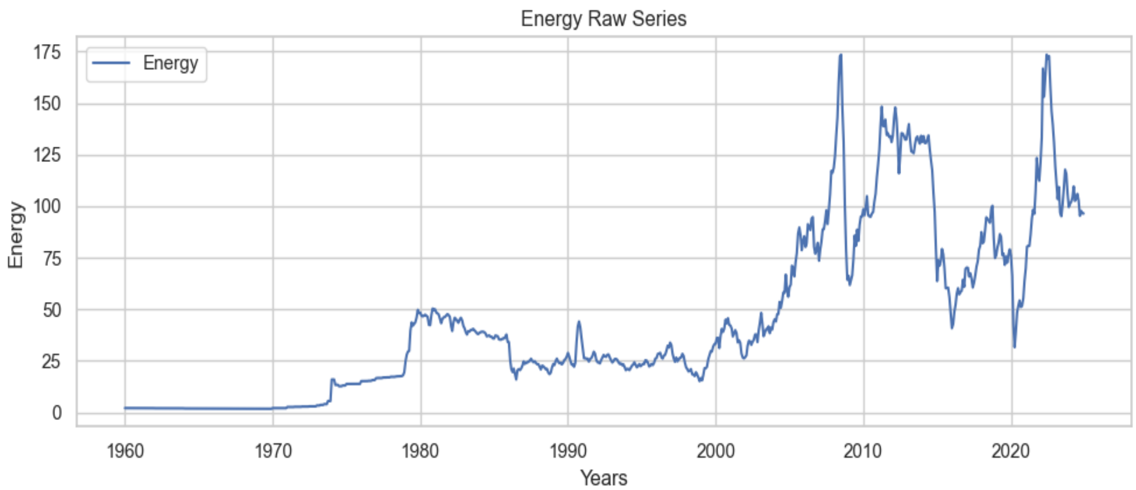

Fig 1. Time plot on Raw data on Nigeria Energy

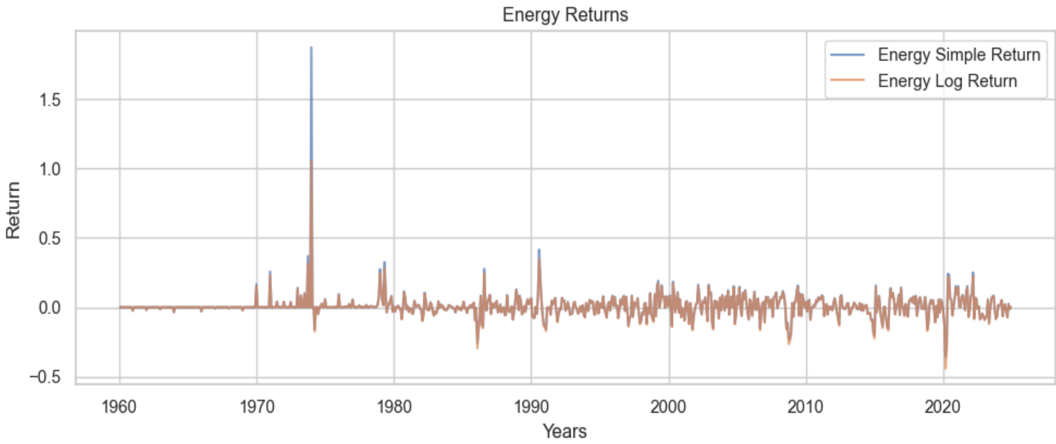

Fig 2. Time plot on simple and logarithm Returns on Nigeria Energy Data

The examination of Nigeria's energy statistics starts with the time series plots and descriptive analysis. In Figure 1, the raw energy data is displayed through a time series graph, providing a visual representation of how energy values change over time. This graph indicates that the raw information shows significant variations and an evident non-stationary pattern. The trend reveals moments of rapid increases and decreases, implying the existence of structural changes or disruptions, possibly resulting from shifts in economic policies, reforms in energy, or fluctuations in the global oil market impacting Nigeria. Figure 2 illustrates the time series plots for both the simple and logarithmic returns of the energy statistics. These return series seem to demonstrate more stationary characteristics, oscillating around an approximate mean of zero. Nonetheless, a visual examination uncovers abrupt spikes, highlighting the existence of volatility clustering and possible leptokurtosis—traits often found in financial and energy return series.

Table 1. Descriptive Statistics on Raw, simple and logarithm Returns on Nigeria Energy Data

Statistic	Energy (Raw)	Energy (Simple Return)	Energy (Log Return)
Mean	44.6334	0.0082	0.0049
Median	29.7985	0.0000	0.0000
Std Dev	41.2843	0.0954	0.0775
Skewness	1.0555	9.7726	2.9824
Kurtosis	0.2405	188.6389	46.1286
Jarque-Bera	146.0189	1152523.4676	69311.7062
JB p-value	0.0000	0.0000	0.0000

The data presented in Table 1 corroborate and quantify these visual observations. The average of the raw energy data stands at 44.6334, while the median is recorded at 29.7985, indicating a positively skewed distribution. This skewness is affirmed by a value of 1.0555. The standard deviation for the raw data is 41.2843, signifying elevated volatility, which is characteristic of raw price or consumption metrics where upward trends and macroeconomic variations have a crucial influence. When assessing returns, both the simple and log return series reveal means that are nearly zero (0.0082 and 0.0049 respectively), accompanied by medians of zero, which aligns with the typical behavior seen in return series. However, the simple return series exhibits a considerably larger standard deviation (0.0954) in comparison to the log returns (0.0775), indicating that the logarithmic transformation is more adept at limiting extreme values. The most notable aspects in Table 1 are the skewness and kurtosis figures pertaining to the return series. Simple returns display an extreme skewness of 9.7726 and a kurtosis of 188.6389, while log returns present a skewness of 2.9824 and a kurtosis of 46.1286. These statistics imply that both series deviate significantly from normality, exhibiting extreme right-tail behavior and frequent outliers. The results of the Jarque-Bera test back up this finding, producing exceedingly high statistics (for instance, beyond 1.15 million for simple returns) alongside p-values of 0.0000, which strongly dismiss the null hypothesis of normal distribution.

The significance of these results is considerable. Initially, the pronounced skewness and kurtosis within the return series indicate that risk is not evenly distributed. Investors and policymakers must acknowledge that the likelihood of extreme events, especially large positive returns, is greater than what would be anticipated under a normal distribution. This observation has immediate repercussions for strategies related to risk management, portfolio development, and pricing in the energy sector. Furthermore, the noticed volatility clustering and non-normal behavior necessitate employing advanced econometric models such as GARCH, EGARCH, or stochastic volatility frameworks capable of addressing heavy tails and temporal volatility variations. These insights align with, and at times broaden, existing research. For instance, Carnero, León, and Ñíguez (2023) explore energy returns and discover significant skewness and kurtosis in energy prices, although they generally report negative skewness, reflecting on the risks associated with downturns. The observed positive skewness in the energy returns of Nigeria diverges from the expected pattern, potentially due to specific market influences like energy subsidies, supply disruptions, or political actions that lead to significant price increases. In a similar vein, Chen, Li, and Worthington (2020) indicate that the skewness noted in returns from U.S. industries, including energy, serves as a valuable indicator for predicting future returns, while kurtosis offers lesser forecasting ability.

Their findings underscore the significance of skewness as a risk element, a trend also evident in Nigerian data, where pronounced positive skewness could suggest the possibility of lucrative return premiums. Gabrielsen et al. (2012) stress the necessity of considering the changing nature of higher moments—variance, skewness, and kurtosis—in Value-at-Risk assessments. This methodology would be particularly applicable to the energy returns in Nigeria, given that static models do not account for the evolving nature of risk.In a related investigation, Baruník and Kurka (2021) analyze the continuity of higher moments concerning asset returns and advocate for a varied approach to modeling volatility, skewness, and kurtosis. Due to the notable skewness and kurtosis in Nigeria's energy returns, employing a dynamic and varied modeling strategy would prove especially advantageous. These results are consistent with the observations made by Lai (2012), who highlighted the crucial functions of skewness and kurtosis in making hedging choices, especially in energy markets characterized by frequent price spikes. The significant kurtosis present in Nigeria's energy return series emphasizes the necessity of addressing tail risks in both investment strategies and policy decisions. The raw and return datasets from Nigeria's energy sector exhibit trends like those identified in international energy markets, particularly regarding non-normal distributions and volatility. Nonetheless, the degree of skewness and kurtosis—particularly the pronounced positive skew—sets Nigeria apart, likely attributable to its distinct market framework and external shocks.

These insights underline the urgent need for risk management tools and modeling methods tailored to the specific context, moving past conventional assumptions.Also, identifying these breaks is essential for precise modeling and prediction, since overlooking them might result in false interpretations. The table below shows the computed exact dates on which structural interruptions were identified through the ruptures technique, emphasizing times that might relate to significant historical, economic, or policy-driven occurrences influencing the energy sector.

Table 2 . Structural Break Detection (ruptures) in Nigeria Energy Data

Series	Detected Break Dates
Energy	1973-09-01, 1979-02-01, 1985-10-01, 1999-12-01, 2004-12-01, 2010-10-01, 2014-12-01, 2021-03-01

Table 2 identifies eight specific dates where structural breaks were identified in the Energy series: 1973 09 01, 1979 02 01, 1985 10 01, 1999 12 01, 2004 12 01, 2010 10 01, 2014 12 01, and 2021 03 01. The time plot (Figure 3: Energy Structural Break) serves as the time-series visualization of the energy variable throughout the analyzed period, with vertical indicators marking each of the identified breakpoints. Collectively, the table and graph illustrate when shifts in regimes occur and depict how the behavior of the energy series changes before and following each structural break. The identification of structural breaks on these dates indicates that notable regime changes transpired within Nigeria’s energy sector or its wider economic landscape, impacting the energy series substantially. Some probable connections to developments in Nigeria include the break on 1973 09 aligning with the global oil crisis of 1973 and the increasing oil income in Nigeria. During this period, Nigeria began to vigorously utilize its oil revenues for developmental initiatives and social programs (Ighravwe, Ajenifuja, & Usman, 2022).

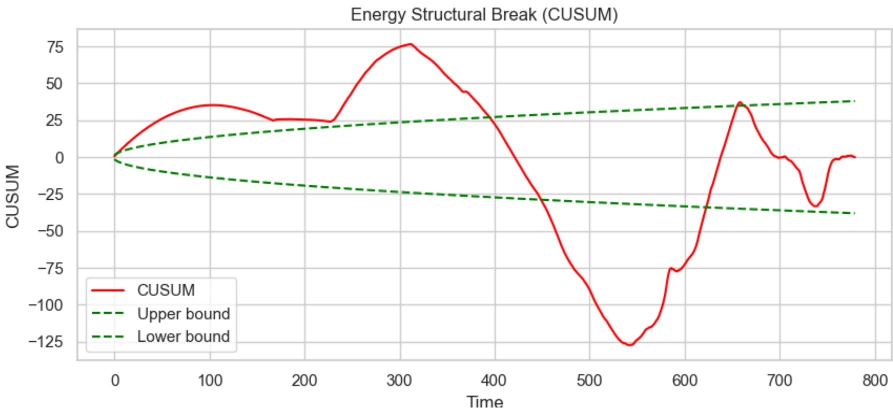

Fig 3. Structural Break in Nigeria Energy Data

The disruption occurring in 1979 may be linked to institutional changes: during that time, the Energy Commission of Nigeria was formed under Act No. 62 of 1979 to oversee the country's energy policies (Energy Commission of Nigeria, 2025). The disruption seen in October 1985 could be a consequence of the oil price decline in the 1980s and the structural adjustment policies applied in Nigeria (for instance, the adjustments made under SAP) that influenced energy investments, subsidies, and demand levels. The disruption in December 1999 comes shortly after the establishment of democratic rule in 1999 and the initiation of reforms in the power sector, which involved the restructuring and breaking up of NEPA into PHCN as well as the introduction of new regulatory systems (Galadima, & Aminu, 2019). The break in December 2004 is likely connected to ongoing reforms, such as the deliberations and implementation of the Electricity Power Sector Reform (EPSR) initiatives at that time (Ighravwe et al. 2022). The disruption in October 2010 seems to be related to the reforms and investment opportunities in generation, a rise in gas usage, and alterations in policy focus (Galadima, & Aminu, 2019). The December 2014 break may illustrate the effects of the international oil price drop that year, which severely impacted Nigeria's oil revenue and, consequently, energy investment and demand trends. The break in March 2021 probably aligns with the recent enactment of the Petroleum Industry Act (PIA) in 2021, which reformed the oil and gas industry, enhanced regulatory transparency, and changed institutional frameworks (Lee, & Chang, 2005).

Many of these disruption points coincide with significant policy reforms, external shocks (such as global oil cycles), or changes in regulation and institutional structure within Nigeria's energy sector.The energy series is dependent on the regime in place and shows structural instability, indicating that any modeling (for example, regression, cointegration, or causality) that overlooks these breaks runs the risk of misinterpretation. Predictive analyses might lack reliability during times of structural disruption. Policy evaluations must recognize that the impact of various factors may vary across different regimes. The findings also indicate that Nigeria's energy sector responds to both external and internal shocks, highlighting the necessity for adaptive and resilient policy frameworks. The presence of recent breaks (2014, 2021) suggests that the sector continues to develop amid new challenges (such as energy transition and regulatory reforms). Prior empirical investigations concerning energy series often identify structural breaks and underscore their significance. For example, Lee and Chang (2005) discovered structural breaks in the relationship between energy and economic growth, showing that accommodating breaks changed the causality results. Research conducted on Iran (utilizing Zivot-Andrews and Gregory-Hansen tests) identified breaks in energy consumption patterns and confirmed long-term links between energy usage and economic growth.

A study in Mexico spanning from 1965 to 2014 revealed two structural breaks after which energy consumption began to lead to GDP performance. In Nigeria, Galadima and Aminu (2019) identified structural breaks in the consumption of natural gas and its growth, demonstrating that neglecting these breaks results in biased conclusions. In the realm of power sector reforms, Ighravwe et al. (2022) documented how structural and institutional shifts in Nigeria's electricity sector have transformed its capacity and effectiveness. These findings are consistent with yours: structural breaks are widespread and significantly influence interpretations.

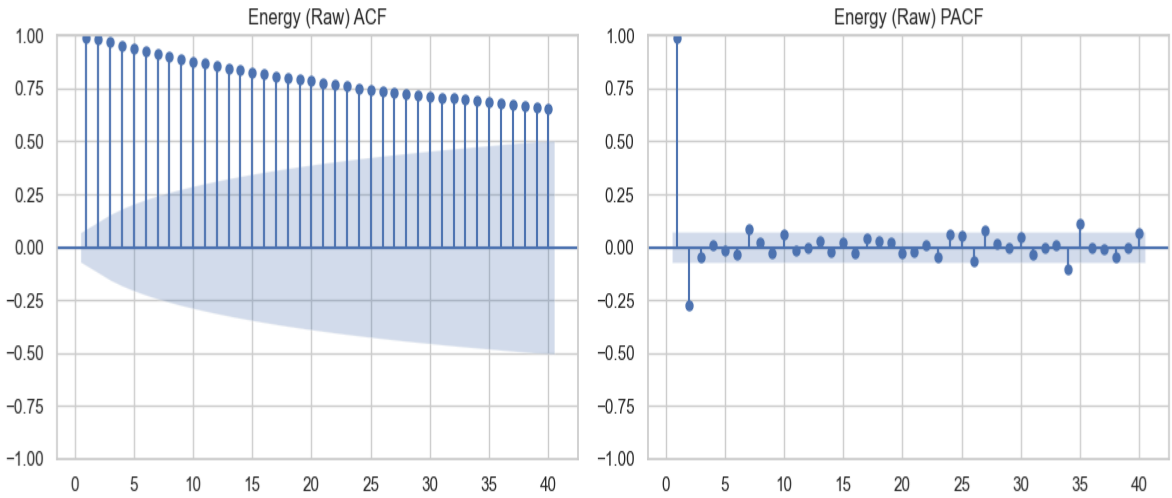

Fig 4. ACF and PACF for Raw on Nigeria Energy Data

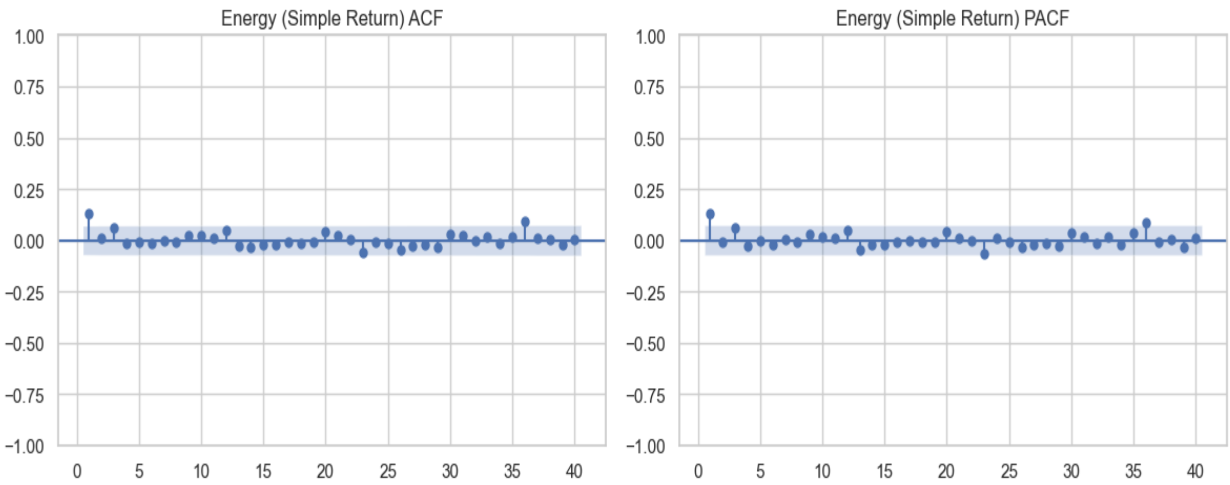

Fig 5. ACF and PACF for simple Returns on Nigeria Energy Data

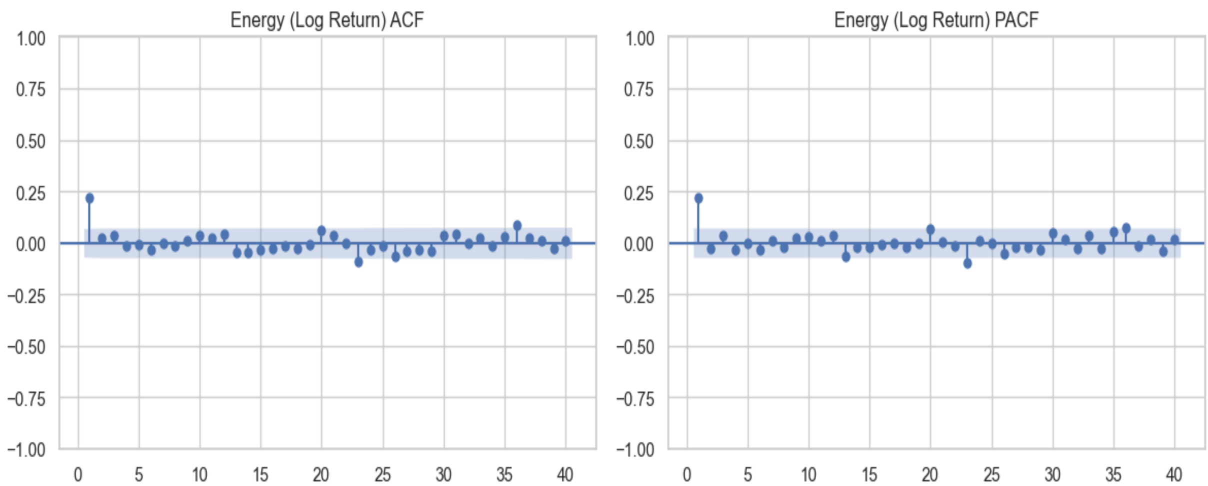

Fig 6. ACF and PACF for logarithm Returns on Nigeria Energy Data

Figures 4, 5, and 6 illustrate the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF), which are instrumental in recognizing serial dependence and selecting models in the analysis of time series data. The raw energy price data (Figure 4) is probably non-stationary, making it inappropriate for direct application of ARMA techniques. The gradual decline in ACF indicates the presence of unit root characteristics. Both simple returns (Figure 5) and log returns (Figure 6) are expected to be stationary, as their ACFs and PACFs show rapid decreases, suggesting that the differencing method has effectively addressed non-stationarity. In the case of returns or log-returns, the ACF and PACF truncate after lag 1 and 2, suggesting that a low-order ARMA model (for instance, AR (1), MA (1), or ARMA (1,1)) could be fitting. This leads to the important conclusion that one should refrain from using the raw energy data for modeling and forecasting due to its non-stationary nature. Employing stationary transformations such as simple or log returns is deemed more suitable. The decision between using simple or log returns is based on the characteristics of the energy variable—log returns are typically employed for price data, whereas simple returns may be applicable for quantity data. These insights are crucial for forecasting time series, advocating for pre-whitening through differencing prior to ARFIMA modeling. The residuals obtained from the returns suggest further autocorrelation when squared; therefore, GARCH-type models may be necessary. Any regression analysis involving the energy series must consider stationarity and structural changes to prevent producing spurious outcomes. The autocorrelation patterns observed, especially within the raw data, reinforce the indication of non-stationarity and possible regime shifts, as recorded in Table 2. Comparable observations have been made in studies related to energy in Nigeria (for instance, Galadima & Aminu, 2019), where energy series necessitated differencing and alteration for structural breaks to develop reliable models.

Table 3. Unit Root Tests in Nigeria Energy Data

Test Statistic	Energy (Raw)	Energy (Simple Return)	Energy (Log Return)
ADF Statistic	-1.9338	-24.4013	-22.3664
ADF p-value	0.3163	0.0000	0.0000
ADF 1% Crit	-3.4388	-3.4388	-3.4388
ADF 5% Crit	-2.8653	-2.8653	-2.8653
ADF 10% Crit	-2.5688	-2.5688	-2.5688
KPSS Statistic	3.2216	0.0956	0.1013
KPSS p-value	0.0100	0.1000	0.1000
KPSS 1% Crit	0.7390	0.7390	0.7390
KPSS 5% Crit	0.4630	0.4630	0.4630
KPSS 10% Crit	0.3470	0.3470	0.3470
PP Statistic	-1.7823	-24.4458	-22.1721
PP p-value	0.3893	0.0000	0.0000

The reports in Tables 3 and 4 deliver a thorough examination of the time series characteristics of energy data—considering aspects like stationarity, conditional heteroskedasticity (ARCH effects), and long memory. The Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and KPSS tests are used collectively to evaluate the stationarity of the energy series. Regarding the raw energy data, the ADF statistic (−1.9338) and the PP statistic (−1.7823) do not yield statistically significant results (p-values exceeding 0.05) and thus do not reject the null hypothesis of a unit root. Conversely, the KPSS test shows high statistics (3.2216) accompanied by a p-value of 0.01, suggesting a rejection of stationarity. As a result, the raw energy series is confirmed to be non-stationary. In comparison, both simple returns and log returns firmly reject the existence of unit roots according to the ADF and PP tests (p-values = 0.0000), while the KPSS statistics (0.0956 and 0.1013) remain below the 10% critical threshold. Therefore, both return series are indeed stationary. This validates that applying first-differencing to the data (as represented in Figures 5 and 6) alters the energy series into a format appropriate for time series modeling. The energy series in Nigeria presents characteristics of non-stationarity, volatility, and persistence, reflecting the historical dynamics of the energy sector, which has faced challenges such as price controls, regulatory shifts, underinvestment in infrastructure, and external disturbances (like oil market crashes). Attempting to model such a series without considering its non-stationary and volatile aspects would likely result in misleading regression outcomes and erroneous forecasts.

Table 4. Results for the test of Presence of ARCH effects in Nigeria Energy Data

Lags	**Raw**	**Simple**	**Log Returns**
5	755.9098[0.000]	1.4407 [0.9198]	9.4044 [0.0940]
10	751.6456[0.000]	1.4611[0.9991]	9.4087[0.4938]
15	746.8962 [0.000]	1.4611[1.0000]	9.4087[0.8472]

ARCH Effects Analysis in Table 4 reveals that for the raw data series, across all specified lags (5, 10, 15), the test statistics are considerable, and p-values are 0.000, indicating a significant presence of ARCH effects. This suggests that the variance of energy levels varies over time, which is a typical characteristic in energy datasets associated with market disruptions, pricing regulations, or shifts in demand. In the case of both simple and log returns, the ARCH tests produce elevated p-values (> 0.05), particularly for the simple returns (p > 0.9 across all lags), signifying a lack of significant ARCH effects. Log returns demonstrate a slight ARCH presence only at lag 5 (p = 0.094), but this is not the case for the greater lags. This indicates that GARCH-type models for volatility modeling may be suitable for the raw energy series, but not essential for the return series unless there is evidence of higher-order clustering of volatility.

Table 5. Tests of Long Memory in Nigeria Energy Data

Test Statistic	Energy (Raw)	Energy (Simple Return)	Energy (Log Return)
Hurst Exponent	0.8551	0.5671	0.5725
DFA Alpha	1.4438	0.5877	0.6158
Rescaled Range (R/S)	0.8551	0.5671	0.5859
Variance Ratio Stat	3.6919	-1.4937	-2.3890
Variance Ratio p-value	0.0002	0.1353	0.0169

Long Memory Properties can be observed in Table 5. The presence of long memory in a time series denotes enduring correlations over extended periods. For the raw energy data, both the Hurst exponent (0.8551) and DFA Alpha (1.4438) significantly exceed 0.5 and 1, respectively, suggesting a strong long memory and possible non-stationarity. The R/S statistics further corroborate the Hurst result, enhancing this inference. The variance ratio test has a statistically significant result (p = 0.0002), signifying that reversion does not apply here likely due to the influence of long memory. As for simple and log returns, the Hurst exponents (~0.57) and DFA Alpha (~0.60) are close to 0.5, which points to weak or absent long memory. The variance ratio for log returns shows statistical significance (p = 0.0169), indicating some tendency toward mean-reverting behavior, while the simple return series does not reveal significant deviations from randomness. The Machine Learning-Enhanced Long Memory Volatility Models in Nigeria Energy Series were estimated, and the results are shown in Table 5 below.

Table 6. Machine Learning-Enhanced Long Memory Volatility Models in Nigeria Energy Series

D	p	Q	Aic	Arfima ->Ann Mse	Arfima ->Svr Mse	Figarch Status	Figarch Ann Mse	Figarch Svr -Mse	Hygarch Status	Hygarch Lambda	Hygarch Ann-Mse	Hygarch Svr-Mse
0.57	2	1	4440.01	0.034	0.042	trained	0.035	0.041	trained	1	65.112	56.546

Table 6 details the findings from long memory volatility models augmented by machine learning techniques applied to energy data from Nigeria, integrating conventional econometric approaches (ARFIMA, FIGARCH, HYGARCH) with machine learning strategies like Artificial Neural Networks (ANN) and Support Vector Regression (SVR). The ARFIMA model, which stands for Autoregressive Fractionally Integrated Moving Average, indicates a differencing parameter (d = 0.57) that verifies the existence of long memory within the energy data, aligning with the outcomes presented in Table 4. The chosen order of the model (p = 2, q = 1) alongside an AIC of 4440.01 reflects a relatively strong fit for the ARFIMA foundation. When improved with ANN, the Mean Squared Error (MSE) decreases to 0.034, while the SVR adjustments result in an MSE of 0.042, indicating that machine learning greatly enhances the predictive accuracy of the ARFIMA model, with ANN achieving a slight edge over SVR in this context.In the case of the FIGARCH model (Fractionally Integrated GARCH), its classification as trained implies a long memory volatility model has been estimated successfully. The FIGARCH combined with ANN produces an MSE of 0.035, whereas the FIGARCH paired with SVR results in a marginally higher MSE of 0.041, reaffirming that ANN is more adept at identifying nonlinear volatility trends in the Nigerian energy context. The HYGARCH model (Hyperbolic GARCH) expands upon FIGARCH by introducing hyperbolic decay in volatility persistence, enhancing its adaptability.

The HYGARCH's Lambda parameter equals 1, indicating a significant persistence in volatility. When enhanced with ANN, the HYGARCH model yields an MSE of 65.112, while that with SVR shows an MSE of 56.546. These values are considerably greater than those from the ARFIMA and FIGARCH models, suggesting inferior performance or a likelihood of overfitting. A notable takeaway is that traditional long memory models significantly gain from machine learning integration: the hybrid models ARFIMA-ANN and FIGARCH-ANN display the highest levels of efficacy, signifying that the merger of linear memory structures with nonlinear pattern recognition results in enhanced forecasting capabilities. The performance of HYGARCH models appears less robust in this instance, indicating that despite their theoretical versatility, they might not be as resilient or might necessitate further adjustments based on the available data. The consistent advantage of ANN over SVR across all hybrid scenarios points to the superiority of neural networks in grasping the nonlinear, regime-dependent dynamics of volatility in Nigeria's energy sector.From a policy and investment viewpoint, precise modeling of volatility facilitates improved risk management, energy pricing forecasts, and infrastructure development planning, particularly in contexts like Nigeria, where shifts in policy and external disruptions (such as removal of subsidies, reforms, and fluctuations in global oil prices) create volatility clusters.

These findings align with broader literature trends: research by Chong et al. (2017) indicates that ANNs surpass traditional GARCH models in predicting electricity volatility in developing markets. Shahzad et al. (2020) show that hybrid ARFIMA–ANN models more accurately capture long-memory dynamics compared to standalone ARFIMA or ARIMA models. Fatai et al. (2023) highlights the success of FIGARCH-type models in addressing oil price volatility across Africa, especially when enhanced by machine learning techniques. In the Nigerian context, Ighravwe et al. (2022) advocate for sophisticated modeling approaches in the electricity sector, citing ongoing volatility and structural transformations.

Table 7. Results of the ARFIMA (2,0.5,1) Model Estimation and Hybrid Machine Learning Performance

Parameters	Coefficients	p-value	
AR (1)	-0.190	0.000	
AR (2)	0.708	0.000	
D	0.57		
MA (1)	0.976	0.000	
AIC	4440.01		
Hybrid MSEs:			
ARFIMA->ANN	ARFIMA->SVR		
0.0344	0.042		ANN status=trained; SVR status=trained

The findings detailed in Table 7 illustrate the parameter estimates from the ARFIMA (AutoRegressive Fractionally Integrated Moving Average) model along with the performance metrics of hybrid models that integrate ARFIMA with machine learning approaches like ANN and SVR. The parameters of the ARFIMA model reveal that the time series demonstrates characteristics of both short- and long-range memory. The notable autoregressive coefficients—AR(1) = -0.190 and AR(2) = 0.708 (both statistically significant with p-values = 0.000)—indicate the presence of substantial lagged relationships within the energy data. Additionally, the moving average term MA(1) = 0.976 (also highly significant) reinforces the role of prior error terms in effectively modeling the series. Particularly important, the differencing parameter D = 0.57 indicates long-memory traits, suggesting that disturbances to the series tend to have a lasting effect, a typical behavior observed in data from energy markets.

The AIC value of 4440.01 serves as a reference point for assessing the model's fit and facilitating comparisons.In the section pertaining to hybrid modeling, the ARFIMA model was combined with two machine learning algorithms-ANN (Artificial Neural Network) and SVR (Support Vector Regression)- to improve forecasting precision. The hybrid models exhibited strong performance, with ARFIMA→ANN achieving a lower MSE of 0.0344, in contrast to ARFIMA→SVR, which yielded an MSE of 0.042. This indicates that the hybrid model enhanced by ANN was marginally more adept at capturing nonlinear trends in the residuals of the ARFIMA model. Both ANN and SVR models were effectively trained, as indicated by their performance status. These outcomes highlight the effectiveness of merging traditional long-memory models with machine learning methods to more accurately represent complex dynamics in energy data, especially when the series demonstrates both linear and nonlinear characteristics.

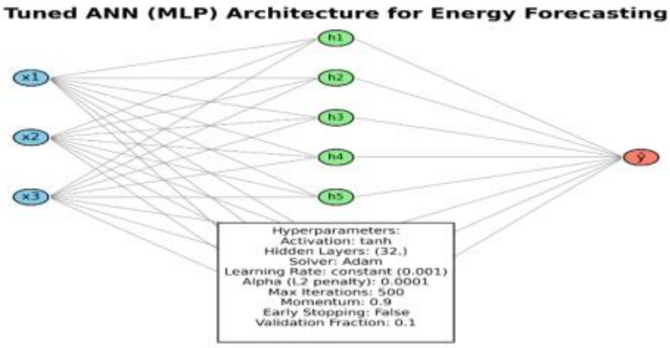

Fig 7. Tuned ANN (MLP) Architecture for Energy Forecasting

Table 8. ARFIMA (2,0.5,1) & HYBRID FORECASTS (12 MONTHS)

Month	ARFIMA Forecast	ANN Forecast	SVR Forecast	Notes
1	0.8874	-8.1912	-3.8076	ANN status=; SVR status=
2	0.5344	0.1841	-0.2490	ANN status=; SVR status=
3	1.3656	-3.7088	-2.0220	ANN status=; SVR status=
4	0.9578	2.5926	1.9617	ANN status=; SVR status=
5	1.6240	-4.6555	-4.6683	ANN status=; SVR status=
6	1.2086	2.8125	9.0367	ANN status=; SVR status=
7	1.7595	0.9672	-0.3487	ANN status=; SVR status=
8	1.3606	2.2365	4.6214	ANN status=; SVR status=
9	1.8266	3.3852	8.7875	ANN status=; SVR status=
10	1.4555	4.6254	4.1958	ANN status=; SVR status=
11	1.8561	1.9665	1.1152	ANN status=; SVR status=
12	1.5172	-2.3574	-2.5440	ANN status=; SVR status=

Table 8 shows the ARFIMA (2,0.5,1) & HYBRID FORECASTS (12 MONTHS) results. The ARFIMA model delivers forecasts that are reliably positive and steady across the span of 12 months, reflecting a more uniform linear trend. On the other hand, the hybrid models, such as ANN and SVR, exhibit higher volatility and broader variations, especially the ANN, which produces numerous extreme negative outputs, implying a heightened responsiveness to nonlinear behaviors or overfitting. In summary, although ARFIMA yields forecasts that are more stable, the hybrid approaches might be reflecting intricate dynamics, albeit sacrificing consistency and ease of understanding.

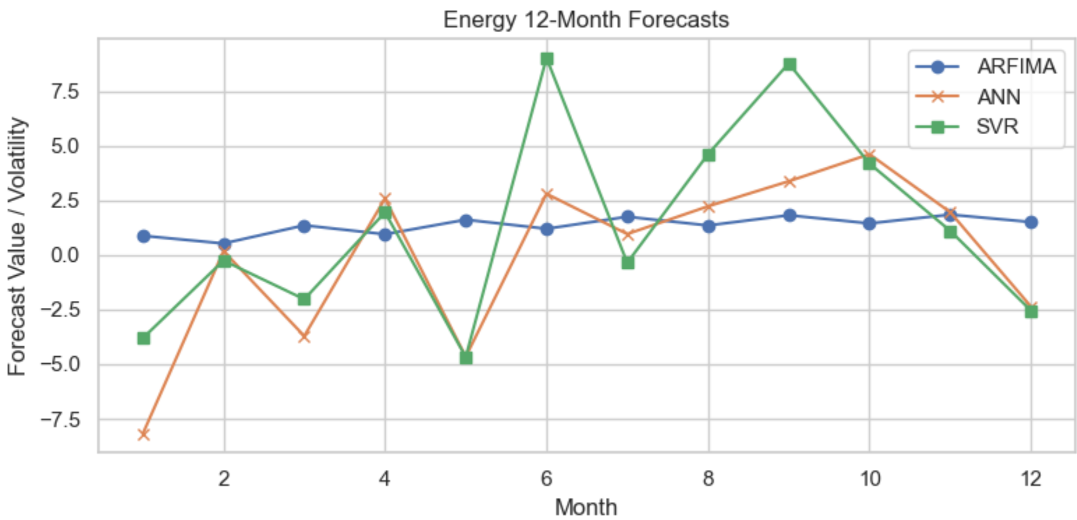

Fig 8. Combine Forecast Plots for ARFIMA, ANN and SVR

Figure 8 illustrates the forecast comparisons for ARFIMA, ANN, and SVR, presenting a visual analysis of the 12-month outlook produced by the ARFIMA (2,0.5,1) model alongside its hybrid models- ANN and SVR. This figure plays a crucial role in evaluating the characteristics, adaptability, and predictive performance of each modeling technique when applied to the Nigerian energy dataset.In the graph, the forecast from ARFIMA manifests as a smooth and gently ascending curve, indicative of the model's long-memory and mean-reverting properties. Such behavior aligns with the anticipated outcomes of ARFIMA's structural formulation, which adeptly captures persistence but is not highly sensitive to short-term fluctuations or abrupt nonlinear shifts. In contrast, the ANN forecast exhibits considerable variability, featuring notable declines (for instance, in Month 1 and Month 5) as well as sharp increases (notably in Months 6 and 9). These variations signify the neural network's responsiveness to nonlinear trends, outliers, or sudden changes in historical data. ANN is adept at grasping intricate temporal relationships that extend beyond simple linear trends, making it beneficial in a tumultuous market such as Nigeria's energy sector. The forecast generated by SVR also displays significant variability, although it tends to be less dramatic than that of ANN. SVR effectively accounts for both short-term fluctuations and broader trends through margin-based learning.

Its performance is characterized by a balanced approach—neither as inflexible as ARFIMA nor as erratic as ANN, though it occasionally deviates by overshooting or undershooting, especially in Months 6 and 9. When the three forecasts are depicted together in Figure 7, the visual distinctions underscore the strengths of each model: ARFIMA serves as a trustworthy reference point, suitable for gradual and stable developments. ANN is proficient at detecting nonlinear and sudden structural transitions, which frequently occur in Nigeria's energy landscape. Additionally, SVR presents a compromise, valuable when volatility exists but remains within manageable limits. This combined representation reinforces insights gleaned from Tables 6 and 7: hybrid models, particularly ARFIMA-ANN, surpass standalone time series models in terms of adaptability and predictive precision. Such visual evidence advocates for energy economists and policymakers in Nigeria to implement ensemble forecasting strategies that merge long-memory characteristics with the adaptability of machine learning, particularly during periods of regulatory shifts or market transformations. This conclusion aligns with previous empirical research. Chong et al. (2017) identified that ANN models are more responsive to fluctuations in energy consumption within emerging markets. Similarly, Shahzad et al. (2020) demonstrated that models integrating ARFIMA and ANN yielded superior forecasting results compared to traditional ARFIMA or SVR alone. In Nigeria, Ighravwe et al. (2022) advocate for the use of adaptive forecasting instruments, given the recent liberalization of the energy market, which has introduced new uncertanties.

IV. CONCLUSION

This research has indicated that the energy market in Nigeria is marked by significant volatility, persistence, and instability, which are shaped by changes in domestic policies as well as external economic factors. The results from the time series analysis indicate that although the energy data in its original form is non-stationary, transforming the returns renders them stationary. This demonstrates that energy prices show long-term dependency and cyclical trends that are characteristic of economies driven by commodities. The identification of eight structural breaks from 1973 to 2021 reveals that the energy sector is influenced by different regimes, which have been affected by global oil crises, institutional changes, and the implementation of initiatives such as the Petroleum Industry Act. The empirical data shows that conventional long memory models, including ARFIMA, FIGARCH, and HYGARCH, effectively represent persistence in volatility. However, their forecasting capabilities see significant enhancement when combined with machine learning techniques. The hybrid models ARFIMA–ANN and FIGARCH–ANN produced the lowest Mean Squared Errors, emphasizing their exceptional forecasting capabilities. Artificial Neural Networks (ANN) surpassed Support Vector Regression (SVR), illustrating ANN's capacity to learn nonlinear and asymmetric trends that exist in Nigeria's energy data.

Hence, the study concludes that merging fractional integration with deep learning models results in more effective and realistic predictions for managing and forecasting energy price volatility. Ultimately, these hybrid methodologies present a sophisticated way to model structural breaks, clustering of volatility, and long-memory phenomena, offering vital resources for predicting, analyzing energy policies, and planning investments in Nigeria's changing energy environment. From a theoretical viewpoint, this study contributes to the econometric field by blending long-memory models with machine learning methods, thus connecting linear statistical techniques with nonlinear computational intelligence. This combination improves the accuracy of models and the reliability of predictions in the unpredictable and structurally fragile energy market of Nigeria.

Also, regarding policy implications, the findings stress the necessity for flexible, data-informed regulatory strategies. Decision-makers can utilize volatility forecasts enhanced by machine learning to foresee shocks, refine subsidy reforms, and stabilize energy prices. Acknowledging structural breaks allows for timely modifications in policies to avert revenue declines and control inflationary pressures stemming from fluctuations in energy prices. For investors and energy planners, the hybrid models present better forecasting reliability, which is crucial for risk management, hedging strategies, and long-term investment in infrastructure. Grasping the dynamics of volatility aids energy companies in accurately assessing risks and strategizing production or importing in uncertain conditions. Similarly, the technology and research standpoint, the outstanding performance of ANN-based hybrid models showcases the potential of artificial intelligence in the field of econometric forecasting. Researchers and analysts are encouraged to investigate other deep learning frameworks—such as LSTM or GRU networks—to enhance long-memory modeling in the domains of finance and energy economics. The fusion of econometric long-memory models with machine learning not only elevates predictive capabilities but also establishes a more resilient system for decision-making regarding energy market volatility management in Nigeria and other developing economies.

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